The Validity of Unidimensional Scales. Different Association Structures in Relevant Subgroups of a Heterogeneous Population

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Abstract

Comparison between groups is one of the main topics of social research. Often, we are not sure if the indicators that are supposed to measure a particular concept have the same meaning in each group of a heterogeneous population. In order to evaluate the equality of meaning between groups we can choose between several methods. In this article five statistical methods for multi-group comparison are presented and critically evaluated. Based on the findings we must conclude that it is necessary to combine different methods because they have several weaknesses and sometimes complementary strengths.

1. Introduction

In the social sciences in general and in the sociological survey research in particular scales are widely used. The vast body of literature on both the methodological and the substantive theory reflects their widespread use.

That great attention is being paid to scale construction is largely due to the many abstract, not directly observable concepts that are used in sociological reasoning. Well-known and central sociological concepts like anomie, social integration, tolerance, authoritarianism, cultural status, individualism, and solidarity are all abstract and, very often complex constructs. Other concepts that seem directly observable at first sight show great complexity upon more thorough examination such as socio-economic status. Scaling is the most popular technique for making these abstract constructs observable.

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Since these concepts are not directly linked to an observable reality, we need a measurement theory that links both the abstract concepts to the indicators used to observe it. Note that a prerequisite for such a measurement theory is a clear and well-developed substantive theory of the concept to be measured. Blalock (1968, 1982) termed the theory that links indicators and concepts an auxiliary theory. Although Blalock and, with him, other measurement specialists stressed the importance of an explicit and well-developed auxiliary theory, most sociological theory and empirical research on abstract concepts has confused “auxiliary” with the unnecessary or the unimportant. Thus, auxiliary theory is only poorly developed and in many respects implicit. This is particularly the case for the very issue of the comparability of measures.

Whenever empirical research is done comparison between groups is one of the main topics – comparison between educational levels, sexes, socio-economic levels, life-world cultures, nations. Recently, longitudinal research has become a focus of social research. This frequent use of comparison of measures in empirical research is not being accompanied by commensurate attention to the comparability of measures either theoretically or methodologically. Sociological methodology in survey research has mainly been concerned with error due to interviewer behaviour (cf. Billiet and Loosveldt, 1984, 1988; Sudman, Bradburn, and Schwartz, 1996) and error due to response communications, memory effects, and other respondent effects. It is somewhat surprising that Sociology is concentrating mainly on individual respondent traits such as the motivation of the respondent, non-attitudes and acquiescence. Social desirability is a more social variable but rarely is it presented as a feature of a social group. Much, and perhaps most, of the effort in sociological methodology is focused on questionnaire design (cf. Schumann and Presser, 1981; Groves, 1989).

In the present article, we will assess the problem of comparability of measures between relevant sub-groups of a heterogeneous population. The question is of great importance. If, for example, anomie is greater in the less educated groups but if the validity of the indicators used to measure anomie differs depending on the educational level, then we do not know what these differences between educational groups really mean. In educational and psychological research, comparability of measures has received much more attention, especially under the impetus of the question of comparability of intelligence tests, which has important ethical and political consequences (cf. Berk, 1989). The absence of such important consequences may be an explanation for the lack of interest of sociological methodology in multi-group comparison. Some procedures that are being applied in educational research will be presented below.

First of all, there is the problem of theory. In almost all substantive theories, there is no developed auxiliary theory linking the indicators to the concepts and dealing with the problem of the items having the same meaning to all sub-groups of a given population. As Blalock points out 'there is no effective way to handle the comparability issue without an explicit auxiliary measurement theory that can pin down the precise nature of potential non-comparability of measures across settings,
time periods or individuals' (Blalock, 1982:60). This substantive theoretical problem is beyond our present scope, which is limited to the presentation and application of some possibilities for multi-group comparison. We do hope that substantive and auxiliary theory will deal with the problem of comparability in the near future.

The central thesis that will be argued in this paper is that the way indicators and latent variable are linked should be the same in the different relevant contexts. These relevant contexts can be differences in educational level, sex, socio-economic status, culture, language, or time. The same stimuli (items) can evoke different consequences depending on the setting in which they are presented and thus create different meanings, even though the concept is supposed to be measured in the same way in each setting. If this is the case, the errors that arise are attributed to construct bias. We will illustrate some possibilities for detecting construct bias in survey research will be illustrated using two well-known sociological concepts: Adorno's authoritarianism and tolerance towards racial minorities. The technical data concerning the population and the measures are discussed in the following section.

2. Population and measures

2.1 Scales

Among the wide variety of scales available today, Adorno's F-Scale (1950) is one of the most popular social measures (Meloen, 1991). Critics like Rorer (1965) and Peabody (1966) have questioned the application of this measurement instrument, but the number of fundamental methodological concerns with this scale are few. Since the publication of Adorno's 'Authoritarian Personality' in 1950, evidence has been gathered to prove the reliability and the validity of the F-Scale. According to some critics, the scale had to be adapted. The result of this process is a wide variety of balanced and unbalanced scales by Rokeach (1960), Kohn, (1972, 1974), Altemeyer, (1981) and others. Every country with a research tradition in political attitudes or personality research has its own variants of the F-scale. Some of the recent, most comprehensive work on authoritarianism has been done by Jos Meloen (1983, 1988, 1991, 1993). After reconsidering the reliability and validity assessment of the authoritarianism measure(s), he argues that there is enough evidence to conclude that the original F-scale is a reliable and valid measure for anti-democratic and fascist tendencies in contemporary society (Meloen, 1991, 1993).

Criterion validation is the most popular technique for providing evidence to assess the validity of authoritarianism scale(e.g., Eysenck, 1954; Christie, 1956; Sherwood, 1966; Orpen, 1970; Steiner and Fahrenberg, 1970; Mantell, 1974 and Meloen, 1988). One of the main shortcomings of the research on the validity of the F-scale however is the use of a homogenous population -almost always students- for validating the instrument.
The second aspect of the criticism and validation of the authoritarianism scale is acquiescence bias (yes saying), but evidence has been reported that supports the utility of the original unbalanced scale (Meloen, 1993). However, in our opinion, the important criticism of Rorer (1965) has not ever been refuted. As the opponents of the F-scale have neglected the results of criterion validation that support its validity, its advocates have ignored Rorer’s objection, which was primarily concerned with the vagueness of the items.

Because its advocates argue that it is both reliable and valid, we used a shortened version of the original F-scale to measure the authoritarianism concept. All the items used were worded in the original way (see Appendix). Six of the seven subdimensions of authoritarianism were included in the scale. The Dutch translation is the one commonly used in current research.

Attitude towards immigrants has been a very topical political and social issue since the emergence of extreme-right parties in Europe. In general, the issue is comparable to authoritarianism in the 1950s and the competing 'anomie' scale was designed to predict racial prejudice. Since the extreme right was gaining popularity in Flanders and because of tensions between immigrants and the autochthonous population, such research was also conducted in Flanders. Attitudes towards immigrants were measured with a balanced 8-item scale (see Appendix) based on the research of Felling et al. (1986, 1987), Billiet et al. (1990), and Waege (1991, 1993). After exploratory research, the scale was adapted to the Flemish situation in the way that immigrants were defined (Billiet, Carton, and Huys, 1990).

2.2 Sample

The data for this paper were obtained from a study on political attitudes of 20- and 21-year old young adults in Flanders (the Dutch speaking part of Belgium). A two-phase cluster sample was used. First, we selected 45 Flemish towns or cities on the basis of a proportional stratification of the degree of urbanization. We then selected from each community a group of post-adolescents by means of random sampling stratified by sex. The names were taken from the official registers of the general elections of 24 November 1991. The respondents were interviewed between 15 February and 20 March 1992. To improve the quality of the results of the survey, the interviewers were trained and the interviewers were checked by both written and telephone requestioning of a sample of the respondent group. Ultimately, 473 usable questionnaires were obtained. There was a drop-out rate of 19.3% from the original sample (refusals, incorrect addresses, movers, etc.). The population can be considered representative of the Flemish 20- and 21-year olds.

The interviewers were trained by the ISPO.
3. Bias in construct validity

Likert scaling is the most popular scaling technique in the social sciences. In order to evaluate the validity of the concept – are all items affected in the same way and to the same extent by the same concept – factor analysis is used. Factor analysis is also the most popular technique in educational research for investigating construct bias. As Reynolds has pointed out, ‘consistent factor analytic results across populations do provide strong evidence that whatever is being measured by the instrument is being measured in the same manner and is in fact the same construct within each group. If factor analytic results (i.e., the interrelationships of the variables comprising the test) are constant across groups then one may have greater confidence that the individuals in each group perceive and interpret the test materials in the same manner’ (Reynolds, 1989:201).

There are two main ways of using factor analytic results to compare between groups. The first way is more qualitative and does not use of over-all statistical significance tests. Here, similarity is the main concern. The second approach does use over-all statistical significance tests between models. Here, statistical significance is the concern. The second approach is almost entirely covered by multi-group comparison with confirmatory factor analysis using structural equation models (cf. Jöreskog 1971). The first approach, with the criterion of similarity, is more flexible. We will present both approaches.

3.1 The factor model and inspection of similarity

If a scale is unidimensional, this means that all items in the scale are very highly correlated, or at least sufficiently correlated, depending on the theoretical point of view. In the latter case, the error variances do not have to be zero or equal (see the congeneric model; Carmines and McIver, 1981). But even in more tolerant models vis-à-vis the underlying latent concepts of scale items (see general factor model; Carmines and McIver, 1981), the items of a unidimensional scale must still have approximately the same association structure. The meaning, measured in our case by the correlation and association structure of the items, should be the same in the different sub-groups of a given population. These sub-groups are formed on the basis of relevant criteria or assumptions. We interrogated the scale items by means of a check on dimensionality in an effort to determine whether or not they conformed to the same basic rule of association for a unidimensional scale regardless of any subgroup membership within a population. (cf. Osterlind, 1983). Four central criteria for dividing a population at a certain place at a certain time into relevant sub-groups are age, sex, ethnicity, and education. Since our data are obtained from an age- and ethnic-homogenous population and because we must limit the scope of this
article, Adorno's scale and the attitude towards immigrants scale will be tested only for the different educational groups.

To examine this problem, we divided the original sample of 473 respondents into 4 different groups based on educational level.

<table>
<thead>
<tr>
<th></th>
<th>Group Description</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>least</td>
<td>1st group: vocational education, junior secondary technical education</td>
<td>100 respondents</td>
</tr>
<tr>
<td></td>
<td>2nd group: humanities, secondary technical education</td>
<td>145 respondents</td>
</tr>
<tr>
<td></td>
<td>3rd group: short-term higher non-universitary education</td>
<td>120 respondents</td>
</tr>
<tr>
<td></td>
<td>(respondents still studying)</td>
<td></td>
</tr>
<tr>
<td>most</td>
<td>4th group: long-term higher non-universitary education and university education (respondents still studying)</td>
<td>108 respondents</td>
</tr>
</tbody>
</table>

The first step is to determine the number of factors to retain. Since the theory states that all the items are indicators of one concept, that is 'authoritarianism', we can use confirmatory analysis. Since we are only expecting one factor, exploratory factor analysis could also be used, but the linear structural approach has several advantages. First of all, an over-all test statistic is available that provides us a criterion to check if the theoretical model specifying one factor is acceptable. Conventional exploratory factor analysis uses the eigenvalue = 1 criterion, which has proven to be unreliable because it tends to overestimate the number of factors to retain (Zwick and Velicer, 1986). The overall model fit (chi-square) is also useful in the discussion of the number of cases. Because an acceptable number of cases is needed for stable factor-analysis results, it is possible that a model cannot be accepted because the number of cases is too low. Hoelter's critical N provides information to assess this problem. The linear structural relation modelling approach also provides separate significance tests for each of the factor loadings on the factor. Thus, items that prove to be irrelevant for the measurement of the latent variable can be detected. Standardized factor loadings are also provided, so the importance of each loading can be estimated without the interference of different standard deviations. Another important advantage is the possibility of specifying theoretically relevant correlations between the unique part of the items which are called covariances between the error terms. If several factors are expected, there are more advantages in using linear structural relations models (Cf. Long, 1983:11-15). Problems with models for which more than one latent variable is expected are beyond our present concern.

The theoretical factor-analytic model chosen is the less restrictive one. There are no constraints on the factor loadings and no constraints on the error variances. All are free to be estimated. Thus, we do not expect each item to be an equally good indicator for authoritarianism. We specified for each educational group a model with one factor and eleven indicators. If the model with one latent variable has an acceptable over-all fit, the one factor model is accepted. The second step is the evaluation of the factor loadings. In all factor-analysis approaches, the factor loadings are examined to see if the variance of the item determined by the concept is
important enough. The minimum factor loading for an item to be described as a relevant item was set at .45. The more demanding criterion was set at .60. The determination of these criteria is based on an approximation of the practices and criteria applied in many research papers because there is no widely accepted general criterion. The results of all four confirmatory analyses using linear structural equations are presented in Table 13.

Table 1: Authoritarianism scale. Standardized factor loadings (lambdas). The non-significant parameters are printed in italics. The standardized loadings of .45 or higher within each group are printed in bold-face type.

<table>
<thead>
<tr>
<th>Items/Education</th>
<th>Lesser educated</th>
<th>Middle-lower</th>
<th>Middle-higher</th>
<th>Higher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>0.2694</td>
<td>0.1436</td>
<td>0.3338</td>
<td>0.2114</td>
</tr>
<tr>
<td>Item 2</td>
<td>0.6022</td>
<td>0.3426</td>
<td>0.5684</td>
<td>0.3718</td>
</tr>
<tr>
<td>Item 3</td>
<td>0.3525</td>
<td>0.2807</td>
<td>0.5515</td>
<td>0.2162</td>
</tr>
<tr>
<td>Item 4</td>
<td>0.3280</td>
<td>0.4978</td>
<td>0.3858</td>
<td>0.6210</td>
</tr>
<tr>
<td>Item 5</td>
<td>0.4000</td>
<td>0.4422</td>
<td>0.5287</td>
<td>0.4706</td>
</tr>
<tr>
<td>Item 6</td>
<td>0.3364</td>
<td>0.4573</td>
<td>0.4579</td>
<td>0.7003</td>
</tr>
<tr>
<td>Item 7</td>
<td>0.4575</td>
<td>0.2627</td>
<td>0.2905</td>
<td>0.3893</td>
</tr>
<tr>
<td>Item 8</td>
<td>0.3024</td>
<td>0.2688</td>
<td>0.3623</td>
<td>0.3096</td>
</tr>
<tr>
<td>Item 9</td>
<td>0.3329</td>
<td>0.5941</td>
<td>0.5414</td>
<td>0.5721</td>
</tr>
<tr>
<td>Item 10</td>
<td>0.5752</td>
<td>0.5892</td>
<td>0.5383</td>
<td>0.4279</td>
</tr>
<tr>
<td>Item 11</td>
<td>0.4692</td>
<td>0.3394</td>
<td>0.4208</td>
<td>0.2899</td>
</tr>
</tbody>
</table>

| chi-square      | 54.9725         | 60.5565      | 56.9700       | 55.7213 |
| DF              | 44              | 44           | 44            | 44      |

For all four groups, a model with one latent variable can be accepted since the coefficient CHI-SQUARE / DF is less than two (some accept three; Carmines and McIver, 1981; Bollen, 1989:278).

The answer to the second question concerning the equality of factor loadings is less positive. Only three items, two in the most educated group and one in the least educated group, meet the more demanding criterion of a factor loading greater than .60, all the other loadings in all the other groups being less than .60. In the first, second, and fourth groups, there are only four items that meet the criterion of a loading of .45 or greater, in the third group there are six of eleven items that meet the criterion. Only the fifth item has a loading that is above .40 in every group. Except for this item, there is no item that approximates an acceptable loading in all four groups at the same time. Looking for irrelevant items, we see that the first item is irrelevant for both the second and the fourth group and that the third item is irrelevant for the fourth group. In general, we can say that there is evidence from this factor analysis that does not support the generally accepted implicit thesis that

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3Structural equations computed with SAS®, PROC CALIS, using the LINEQS statement. All lambdas and thetas were free to estimate. All covariances between the error terms were fixed at zero. The variance of authoritarianism was fixed at one. Covariances were used as input.
the items assumed to measure authoritarianism behave in approximately the same way in all four groups.

The analysis was also conducted for the attitude-towards-immigrants scale and the results are presented in Table 2. 4

Table 2: Tolerance towards immigrants. Standardized factor loadings (lambdas). There were no non-significant parameters. The standardized loadings less than .60 within each group are printed in bold-faced type.

<table>
<thead>
<tr>
<th>Items/Education</th>
<th>Least educated</th>
<th>Middle-lower</th>
<th>Middle-higher</th>
<th>Higher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>0.7574</td>
<td>0.8755</td>
<td>0.7745</td>
<td>0.7664</td>
</tr>
<tr>
<td>Item 2</td>
<td>-0.7234</td>
<td>-0.6874</td>
<td>-0.5540</td>
<td>-0.7057</td>
</tr>
<tr>
<td>Item 3</td>
<td>0.7290</td>
<td>0.7447</td>
<td>0.7620</td>
<td>0.5774</td>
</tr>
<tr>
<td>Item 4</td>
<td>0.6591</td>
<td>0.7569</td>
<td>0.7008</td>
<td>0.6335</td>
</tr>
<tr>
<td>Item 5</td>
<td>-0.5923</td>
<td>-0.7543</td>
<td>-0.6135</td>
<td>-0.6045</td>
</tr>
<tr>
<td>Item 6</td>
<td>0.6688</td>
<td>0.7704</td>
<td>0.7151</td>
<td>0.6094</td>
</tr>
<tr>
<td>Item 7</td>
<td>0.7562</td>
<td>0.7104</td>
<td>0.7137</td>
<td>0.5982</td>
</tr>
<tr>
<td>Item 8</td>
<td>-0.7608</td>
<td>-0.7993</td>
<td>-0.7218</td>
<td>-0.5763</td>
</tr>
<tr>
<td>DF</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
</tbody>
</table>

First of all, none of the loadings is less than .45 and even the loadings that do not meet the more demanding criterion of .60 have loadings that approach .60. This means that, for each group, all the items are acceptable indicators of tolerance towards immigrants. Group three has a higher chi-square than the other groups, which means that the model is less close to the observed data here than in the other three groups. Perhaps, for this group, other covariances between the error terms should be specified. There is still one covariance between the second and the fifth item that can be specified, since both mention comparable groups of immigrants. This was not done since the effect of such a model change would not result in an important change of the estimated factor loadings. Generally speaking, only in the third group could other model parameters be specified. The residuals confirm that freeing the covariance between the second and the fifth item is the most important covariance between error terms that can be estimated. Also, the factor loadings are generally lower in the most educated group. This means that the relevance of the items as indicators for attitudes vis-a-vis immigrants is less in this, the most educated group. Although there are differences between the factor loadings, we cannot reject the possibility that we are measuring the same concept in all four groups.

This analysis of comparability was based on visual inspection of the factor loadings using the .45 and the .60 criteria for a factor loading to be considered important and for the item to be considered an indicator of the concept. We can

4The model specifications are the same as for the authoritarianism scale. A covariance was specified between the error terms of the third and the eighth variable since they are perfectly opposed to each other.
conclude that the comparability of the authoritarianism scale is very low and problematical and that the comparability of the attitude-towards-immigrants scale is much better since all items have acceptable loadings (approximately .60 or more) in all four groups. This first approach uses the acceptability of all items in all groups as the criterion for comparability. Of course, this method does not take into account differences between a factor loading of .60 and .80, which, depending on the standard error of the differences, can be important. To assess this problem, more quantitative measures are needed to evaluate factorial similarity.

3.2 Descriptive summaries of overall similarity

Especially in the case of the attitude-towards-immigrants scale where the comparability of the measures in not rejected by the 'qualitative' evaluation of the factor loadings but where differences can be observed, it is relevant to our present concern to take a more quantitative look at the data in order to determine the degree of similarity. There are several ways of doing so, but the discussion and presentation of all these methods is far beyond the scope of this article. Cattell (1979:246-270) presents five possible measures for comparison between factors where the items are the same and the subjects are different (see also Katzenmeyer and Stenner, 1977; Mulaik, 1972, Reynolds, 1982). In this article, we will confine ourselves to the presentation of one obvious measure that is not recommended and one that is popular in educational sciences and is recommended by Cattell (1979).

One way to estimate the degree of similarity is by means of of Pearson's coefficient of correlation between the pairs of factor loadings. Because ordinary correlation by taking deviations of the mean creates negative deviations when small loadings are compared with high ones (Cattell, 1979:252) and because ordinary correlations do not take into account differences in variance when the rank order of the factor loadings is equal, the use of Pearson's coefficient of correlation is not recommended.

One popular index for factorial similarity that is not susceptible to the same defects as Pearson's correlation is Burt and Tucker's coefficient of congruence ($r_c$). Since there are some doubts about the use of $r_c$ when the variances for the variables are not constant across groups, it is recommended that a factor analysis based on covariances be conducted to determine if the factor loadings are comparable between the two groups. The $r_c$ is calculated using the following equation (Cattell, 1979:252):

$$r_c = \frac{\sum_i b_{1i}b_{2i}}{\left(\sum_i b_{1i}^2\sum_i b_{2i}^2\right)^{1/2}}$$

Where $b_{1i}$ and $b_{2i}$ are the loadings of variable $q_i$ on the compared factors $F_1$ and $F_2$. 
One problem with this test of similarity is that there is no clear test of significance. In order to evaluate the significance, results of Monte Carlo experiments are used (Cattell, 1979:253-255). We will use the table produced by Schneewind and Cattell (Cattell, 1979:568) since it is more complete than others. The results of the calculation of \( r_c \) for the authoritarianism scale and the attitude towards immigrants scale are given in Table 3.

Table 3: \( r_c \) values for all possible group comparisons for the authoritarianism scale and the attitude-towards-immigrants scale.

<table>
<thead>
<tr>
<th>/Education</th>
<th>E1-E2</th>
<th>E1-E3</th>
<th>E1-E4</th>
<th>E2-E3</th>
<th>E2-E4</th>
<th>E3-E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authoritarianism</td>
<td>0.9285</td>
<td>0.9663</td>
<td>0.9189</td>
<td>0.9591</td>
<td>0.9212</td>
<td>0.9285</td>
</tr>
<tr>
<td>Tolerance</td>
<td>0.9954</td>
<td>0.9981</td>
<td>0.9936</td>
<td>0.9975</td>
<td>0.9953</td>
<td>0.9853</td>
</tr>
</tbody>
</table>

When we look at the results of the analysis, we are confronted with a problem. Schneewind and Cattell (1970) propose a minimum value of .78 with a significance level of .01 to have a match between the scales. Based on this criterion, both the authoritarianism scale and the attitude-towards-immigrants scale meet the similarity criterion. In the case of the authoritarianism scale, this conclusion is in conflict with the conclusion based on the 'qualitative' evaluation of factor loadings. This calls for a better insight into the \( r_c \).

In order to obtain that better insight, a small experiment was set up. The \( r_c \) for fictive factor loadings of a scale measured in two groups was calculated. The factor loadings have the following properties. There are ten loadings: one is high and equal in both groups; six loadings are high in one group and moderate, but too low to be good indicators in the other group; three loadings are too low in both groups.

Table 4: Experiment to illustrate properties of \( r_c \).

<table>
<thead>
<tr>
<th>Items/Group</th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Item 2</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Item 3</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Item 4</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Item 5</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Item 6</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Item 7</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>Item 8</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Item 9</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Item 10</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

This results in an \( r_c \) of 0.8415. This value is higher than the .78 needed to have a sufficient match between the pair of factors. On the basis of the coefficient of congruence, we must conclude that both factors are a matching pair. Although every
social scientist will agree after evaluating factor loadings that these two factors may have much in common but that they are not equal, with the coefficient of congruence, we must conclude that they are invariant across the groups.

The problem is that this test is not able to distinguish between two important causes of similarity: the variance of the factor and the differences in the factor pattern (cf. Pinneau and Newhaus, 1964). The conclusion is that \( r_c \) of itself is insufficient for evaluating the comparability of measures. It seems desirable to use similarity indices that are able to distinguish between variance as a cause of similarity and the factor loading pattern as a cause of similarity. As Cattell argues, 'This method gives "credit" for the two factors matched being close in general size, as distinct from being of the same pattern or rank ordering of loadings' (Cattell, 1979:266). The \( r_c \) is useful as an evaluation of the comparability of measures vis-a-vis the general size.

### 3.3 Non-parametric test of equality of loadings one by one

To meet the critique of Burt and Tucker's coefficient of congruence, I propose a different strategy that stresses the similarity of the factor structure and takes into account the problem of differences due to random errors.

The first and most important problem is the estimation of random error in order to be able to determine if the differences between groups are considerably higher than the differences that may be expected by chance. One important measure often used in statistics to assess the problem of significant differences are z-values. The difference is expressed in terms of the standard deviation of the mean. These z-values can only be used properly if the distribution of the differences between the factor loadings is a normal distribution.

There is no relevant information about the distribution and standard deviation of the differences between the factor loadings in the population. To obtain the necessary information, we have to estimate this difference and find a way of assessing the problem of the distribution of those differences. Recently, bootstrapping has been proposed as a way of accomplishing this (Mooney and Duval, 1993), for it provides a technique to generate an empirical estimate of the sampling distribution. LISREL® 8 (Joreskog and Sörbom, 1993) makes it possible to obtain these samples and to calculate the corresponding factor loadings in a simple way. The following steps were needed to obtain the estimations of standard errors of the differences between the factor loadings of the items.

- First, a number of samples had to be drawn from the original sample of 473 respondents. The sampling proportion is 25% of the original sample since we have four educational groups of comparable size. Since the aim is the estimation of standard errors, the number of estimates should be between 50 and 200 (Mooney and Duval, 1993:11). To estimate one difference between
the factor loadings on a given item, we needed two estimations of the loading of a particular item. To obtain 100 estimates of differences, we needed 200 estimates of factor loadings in a group of 25% of the original sample. In this case, 200 random samples with replacement were drawn from the original sample.

- Second, after drawing the 200 samples, the factor loadings on all items of the scale were calculated for each group. Thus, for each item, 200 factor loadings were calculated.
- Third, 100 differences between factor loadings were calculated.
- Fourth, for each of the items the mean difference and the standard deviation from the mean difference were calculated, and the distribution of loadings was checked for normality.
- Fifth, if a normal distribution was obtained, z-values were calculated to estimate the importance of the difference between factor loadings given the standard deviation of the difference.
- Finally, the importance or significance can be assessed using the distribution of normal probabilities.

The results of these steps for the authoritarianism scale are presented in Table 5. The significance of the differences can be tested by proposing a minimum value of z. With a proposed \( \alpha \)-level of .05 or .10, the critical values of z for absolute values is 1.96 or 1.65. Thus, the z-value can be used as a measure of similarity that takes into account differences due to random error. Higher values represent higher differences: if the values are equal to or greater than a proposed critical value of z, one can infer that the differences are significant and important. Since we have 66 estimations of differences between factor loadings, a minimum z-value of about 2 would seem to be necessary to conclude that the differences between factor loadings are important.

Based on this criterion, Items 1, 5, 7, 8, 10, and 11 are comparable between groups, since there is no significant difference between any of the pairs of education groups. Items 2, 4, 9, and 6 are problematic vis-à-vis the comparability between the groups. Evaluating the number of significantly different items in each comparison shows that the comparison with the most educated seems more problematic and that there are problems of comparison with the first and the third education group.

Compared with the first retained criterion, we have to conclude that the comparability between groups is much better based on the z-value criterion than based on the critical value evaluation. The evaluation of the standard errors of the item differences allows a partial interpretation. Since the mean is approximately 0 for all items, the z-value can be approximated by dividing the differences in factor loadings by the standard error. For all the items, all standard errors are above .10, and five items have standard errors of .15 or higher. This means that the differences for these items have to be at least .30 or more before a significant value (z-value) is reached. The consequence of these findings is that a factor loading of .30, which
The Validity of Unidimensional Scales

would lead to rejection of the item in exploratory factor analysis compared with a factor loading of .59 for the same item, which would lead to acceptance of item as an indicator, is not detected as a significant difference by the z-value criterion when the standard error of differences between factor loadings is high.

Table 5: z-values to test factor loading differences between groups for the authoritarianism scale.

<table>
<thead>
<tr>
<th>G1-G2</th>
<th>G1-G3</th>
<th>G1-G4</th>
<th>G2-G3</th>
<th>G2-G4</th>
<th>G3-G4</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>0.73803</td>
<td>0.35630</td>
<td>0.31651</td>
<td>1.13840</td>
<td>0.37744</td>
<td>0.71689</td>
</tr>
<tr>
<td>Item 2</td>
<td>1.99794</td>
<td>0.22322</td>
<td>1.76844</td>
<td>1.73228</td>
<td>0.18706</td>
<td>1.50278</td>
</tr>
<tr>
<td>Item 3</td>
<td>0.40503</td>
<td>1.24797</td>
<td>0.83247</td>
<td>1.72378</td>
<td>0.35666</td>
<td>2.15121</td>
</tr>
<tr>
<td>Item 4</td>
<td>1.54897</td>
<td>0.62643</td>
<td>2.56378</td>
<td>1.07287</td>
<td>1.16513</td>
<td>2.08767</td>
</tr>
<tr>
<td>Item 5</td>
<td>0.29802</td>
<td>0.89283</td>
<td>0.49331</td>
<td>0.60264</td>
<td>0.20313</td>
<td>0.40735</td>
</tr>
<tr>
<td>Item 6</td>
<td>0.78215</td>
<td>0.78563</td>
<td>2.19335</td>
<td>0.08351</td>
<td>0.49123</td>
<td>1.48775</td>
</tr>
<tr>
<td>Item 7</td>
<td>1.51578</td>
<td>1.29832</td>
<td>0.52549</td>
<td>0.20948</td>
<td>0.98231</td>
<td>0.76485</td>
</tr>
<tr>
<td>Item 8</td>
<td>0.24177</td>
<td>0.44776</td>
<td>0.03501</td>
<td>0.71091</td>
<td>0.29816</td>
<td>0.39137</td>
</tr>
<tr>
<td>Item 9</td>
<td>2.02526</td>
<td>1.62133</td>
<td>1.85664</td>
<td>0.42716</td>
<td>0.19185</td>
<td>0.25853</td>
</tr>
<tr>
<td>Item 10</td>
<td>0.19598</td>
<td>0.41926</td>
<td>1.49570</td>
<td>0.55577</td>
<td>1.63220</td>
<td>1.13591</td>
</tr>
<tr>
<td>Item 11</td>
<td>0.64264</td>
<td>0.11129</td>
<td>0.96575</td>
<td>0.32670</td>
<td>0.11847</td>
<td>0.64982</td>
</tr>
</tbody>
</table>

For the attitudes towards immigrants scale, Items 2, 4, and 7 are comparable between groups and Items 3, 5, and 8 are problematical vis-à-vis the comparison between groups. In this analysis, the comparison between Group 2 and Group 4 seems the most problematical. Half of the items show significant differences in factor loadings. The results of this analysis lead to a different conclusion than did the minimum-factor-loading criterion. Based on this criterion, all the items were accepted as indicators of racial tolerance, but based on the z-value criterion, three items seem problematical and especially the comparison between the most educated and the young people who stopped going to school at 18 years. This clearly illustrates the usefulness of z-values when factor loadings are high in all groups. High factor loadings can also differ significantly between each other. Concerning the analysis of the authoritarianism scale, we must conclude that the authoritarianism scale is better since there are only six values on 66 estimations that differ significantly. In the case of the attitude-towards-immigrants scale 8, values on 48 estimations show significant differences. To a degree, this can be explained because it is a lot easier to reach significance in the case of the attitude-towards-immigrants scale than in the case of the authoritarianism scale because the standard errors are much lower, in the latter than in the former, all standard errors being less than .11 and half of the standard errors being less than .8.
Table 6: z-values to test factor loading differences between groups for the attitude-towards-immigrants scale.

<table>
<thead>
<tr>
<th>Item</th>
<th>G1-G2</th>
<th>G1-G3</th>
<th>G1-G4</th>
<th>G2-G3</th>
<th>G2-G4</th>
<th>G3-G4</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>2.06835</td>
<td>0.28535</td>
<td>0.14236</td>
<td>1.76648</td>
<td>1.90947</td>
<td>0.12647</td>
<td>0.056646</td>
</tr>
<tr>
<td>Item 2</td>
<td>0.45282</td>
<td>1.71919</td>
<td>0.27910</td>
<td>1.37744</td>
<td>0.28479</td>
<td>1.55117</td>
<td>0.10534</td>
</tr>
<tr>
<td>Item 3</td>
<td>0.07615</td>
<td>0.30017</td>
<td>1.83590</td>
<td>0.09687</td>
<td>2.03920</td>
<td>2.26321</td>
<td>0.077227</td>
</tr>
<tr>
<td>Item 4</td>
<td>1.01746</td>
<td>0.36822</td>
<td>0.18189</td>
<td>0.53487</td>
<td>1.31373</td>
<td>0.66449</td>
<td>0.086408</td>
</tr>
<tr>
<td>Item 5</td>
<td>2.14924</td>
<td>0.33773</td>
<td>0.22194</td>
<td>1.87649</td>
<td>1.99228</td>
<td>0.18077</td>
<td>0.077725</td>
</tr>
<tr>
<td>Item 6</td>
<td>1.31673</td>
<td>0.55701</td>
<td>0.73698</td>
<td>0.68066</td>
<td>2.13278</td>
<td>1.37306</td>
<td>0.07279</td>
</tr>
<tr>
<td>Item 7</td>
<td>0.31864</td>
<td>0.28581</td>
<td>1.43508</td>
<td>0.10425</td>
<td>0.97935</td>
<td>1.01219</td>
<td>0.100498</td>
</tr>
<tr>
<td>Item 8</td>
<td>0.48140</td>
<td>0.48671</td>
<td>2.03056</td>
<td>0.89522</td>
<td>2.43907</td>
<td>1.61674</td>
<td>0.094245</td>
</tr>
</tbody>
</table>

Generally speaking, we have to conclude that taking into account standard errors of differences between factor loadings does not cover problems that can be caused by less stable estimations because those less stable estimations lead to higher standard errors, which in turn lead to the necessity of very high differences in order to be able to conclude that differences between factor loadings are significant. High standard errors indicate a high tendency for items to depart from the mean. Since the factor loading is an indicator of the connection between the item and the concept, a high standard error means low reliability. This can also be shown by evaluating simple correlations between the items and the factor. Therefore, the general conclusion is that a higher reliability is needed for z-values to be used appropriately evaluate factorial similarity.

3.4 Overall parametric tests

The second general approach, the test for significant over-all differences across groups instead of evaluating similarity, is represented by the linear structural modelling approach (Jöreskog, 1969, 1971). This recent and very popular way of doing multi-group comparison in order to check for construct bias consists of factor analysis in several populations simultaneously. The aim of this analysis is to see if it is possible to specify a model where the factor loadings are set equal across groups. The chi-square test is then used to determine the acceptability (fit) of the model (Jöreskog, 1971; e.g., Corcoran, 1981; Benson, 1987; Byrne, 1989). First, a restrictive model is tested -for example all factor loadings are constrained to be equal across groups - and then a model is fitted where the factor loadings are free to estimate in each group. If there is a significant improvement in fit, then the second model is preferred to the first model and the conclusion is that the factor structure is non-invariant across groups. All LISREL 8 (Simplis) programs are given in Appendix III.
For the authoritarianism scale, the model with equality constraints on all the factor loadings has a chi-square of 264.93 with 200 degrees of freedom; the model with all the factor loadings free to estimate has a chi-square of 230.98 with 170 degrees of freedom. The difference in chi-square is 33.95 with 30 degrees of freedom. There is no significant improvement in model fit when the factor loadings are free to estimate. Based on these findings, we can conclude that the factor loadings are invariant across groups for the authoritarianism scale (first item omitted).

For the attitude-towards-immigrants scale, the constrained model has a chi-square of 199.37 with 127 degrees of freedom; the model with unconstrained factor loadings (lambdas) has a chi-square of 164.24 with 103 degrees of freedom. The difference in chi-square is 35.13 with 24 degrees of freedom. Freeing the factor loadings does not result in a significant improvement in over-all model fit. Based on this criterion, we have to conclude that the factor is invariant across groups. The scale can be used in the same way for the four educational groups.

Again, poor factor loadings constitute an important problem. Saris and Satorra (1993; also Saris, Sattora, and Sörbom, 1987) show that the power to detect misspecifications of a model depends not only on the sample size but also on the model specification, the lower parameters making it more difficult to detect misspecifications. For the authoritarianism scale, we have both low parameter estimates and small samples. It is possible that the power of the test if too low to detect misspecifications. Also Jöreskog (1993) stresses the importance of theory over statistical models. Many researchers using LISREL report difficulties in distinguishing between competing models. LISREL seems most appropriate for distinguishing between models with bad and good fits, it is less appropriate for distinguishing between competing models with acceptable fits.

4. Discussion and conclusion

In this article, five ways of evaluating the similarity of factors between groups were presented and discussed: a qualitative way using common criteria for items to be retained as indicators for a latent variable, the Pearson's coefficient of correlation between factor loadings, Burt and Tucker's r, z-value computation for each item within each group, and the over-all model fit (chi-square) from a linear structural equation modelling approach. These possibilities are only a sample of the possible methods for evaluating factorial similarity, but most of them are commonly used in educational sciences and psychology (cf. Reynolds, 1982; Byrne, 1989). All these methods have their strengths and their weaknesses.

The first method looks within each group to see if all the items are acceptable as good indicators of the concept. To do this, the critical values for the factor loadings were those commonly used in social sciences. With this criterion, we concluded that comparability was problematical for the authoritarianism scale. Most of the factor
loadings were too low, particularly in the least-educated groups. Measures for the internal consistency (e.g., Cronbach's alpha) would lead to the same conclusion as that based on the evaluation of the size of factor loadings: the internal consistency of the scale is too low within each educational group to conclude that the scale is a reliable measure. The most important weakness of this first criterion is that there is no evaluation of significance of differences between groups when the factor loadings are high, such as .60 or higher, and that there is no criterion to distinguish between random differences and substantive differences.

To meet these two important shortcomings of the more qualitative evaluations, the use of z-values was proposed when the differences between factor loadings are normal distributions. To estimate the standard error and the mean for the differences between factor loadings, bootstrapping was used. Since all the differences between factor loadings were normal distributions, z-values could be used. Based on this criterion, the attitude-towards-immigrants scale showed more problems than did the authoritarianism scale. This difference vis-à-vis the first way of assessing factorial similarity was mainly due to the large difference in standard deviations, the standard deviations for the authoritarianism scale being high because of the poor consistency of the scale. So the problem at the first level, detected by simple inspection of the factor loadings for each group, was no longer detected by z-values. On the contrary, high standard deviations for less stable factor loadings for the items lead to the necessity of very high differences. The usefulness of z-values depends on the consistency of the measurement. When the consistency is too low, z-values are less appropriate. When the internal consistency was high enough, as in the case of the racial attitude towards immigrants scale, z-values were useful for detecting an important difference between the most educated and the secondary-school educated young people. Half of the items showed significant differences. This problem recurs in the structural modelling approach.

Within the same context two other measures for factorial similarity were discussed. Pearson's coefficients of correlation between factor loadings was considered on the basis of the literature. Because ordinary correlation takes deviations of the mean, negative deviations are computed when small loadings are compared with high ones. Also ordinary correlation does not take into account differences in variance when the rank orders of the factor loadings are equal. Based on these two important shortcomings, the use of the Pearson's coefficient of correlation was not recommended.

Another, popular measure of factorial similarity is Burt and Tucker's coefficient of congruence ($r_c$). The coefficient was lower for the authoritarianism scale than for the racial attitude towards immigrants scale. This result was not concordant with both the critical-factor loading criterion and the z-value evaluation. The main weaknesses of this coefficient are that there is no way of distinguishing between differences in variance between factors and differences in the factor-loading pattern. The occurrence of high factor loadings in both groups for different items, particularly when these are combined with moderate loadings in the other groups can still lead to
significant values for $r_e$. Although, this coefficient is not useless, the weight of high factor loadings in the total score is very high, too high to obtain a differentiated evaluation of factorial similarity. In this last respect, it is similar to the first way of assessing factorial similarity.

The fifth way of assessing the question of factorial similarity involves LISREL and similar techniques. LISREL was used to specify two different models, one with all factor loadings set equal and one with all factor loadings free to estimate within each group. The differences in chi-square relative to the difference of the degrees of freedom were evaluated. For both scales, there was no significant improvement in model fit when the factor loadings were freed. On the basis of the parsimony criterion, the model with the loadings set equal to each other was accepted. So the LISREL approach did not account for the differences found when evaluating z-values for the attitude towards immigrants scale. These differences disappeared in the high number of equalities. Also, as expected given the kind of analysis, it did not indicate that the authoritarianism scale only had poor factor loadings within each group (combined with approximately equal variances) making it very hard, also for LISREL to detect significant differences between groups especially, when combined with small samples (cf. Saris and Satorra, 1993). Apart from this, it should be noted that the experience of many researchers indicates that LISREL has problems with detecting differences between competing models when differences in over-all model fit are evaluated. LISREL offers possibilities of distinguishing between models with a bad fit and models with a good fit, but LISREL is less appropriate for the ‘exploratory’ evaluation of nearly equivalent models (Jöreskog, 1993). When no theory is available, the parsimony criterion can be applied. When theory is available and LISREL is not able to distinguish between a model with fixed factor loadings and a model with freed factor loadings, we cannot conclude validation or falsification of the theory.

With this last remark, we return to the introduction of this article. An important and hitherto too often neglected concern of social scientists working with measures should be a theory about the comparability of measures between groups. Based on this theory, ways could be developed to evaluate the comparability of measures. From the comparison of five techniques for the evaluation of factorial similarity and a partial discussion of these techniques, we can at least conclude that several techniques have to be applied since all of them have strengths and weaknesses and none was able to distinguish between several aspects of factorial similarity. Therefore, a conclusion that factors are similar or dissimilar across groups based on only one of these criteria should be approached very critically.

To illustrate the possibilities of joint use of several indices for factorial similarity, the conclusions for the two substantive measures used as illustrations are as follows:

- Concerning the authoritarianism scale, we have to conclude that it is very difficult to assess the problem of comparability of the measure since the consistency of the scale is low which creates high standard errors; for LISREL
too, poor loadings can lead to low power of the test. It seems that assessment of the comparability of this measure can only be addressed with more stable and larger estimations of the factor loadings or with larger samples.

- For the attitude-towards-immigrants scale, the consistency of the scale is acceptable. Based on the critical factor loading criterion and r<sub>c</sub>, we can conclude that the measure is comparable across groups. Comparison between LISREL models also leads to the same conclusion. Only the z-values qualify these findings. From the evaluation of z-values, we detect a problem of comparability between the most educated and the secondary-school educated. The items of the scale show less relevance for the most educated. Although the first three criteria show that this problem is not important enough to reject the scale because of non-comparability, the evaluation of z-values shows that the measure for attitudes towards immigrants could be improved with respect to greater relevance for the most educated.

References


The Validity of Unidimensional Scales


**Appendix I**

**Authoritarianism scale**

*Between brackets the original number of Adorno et al., 1950: 255-257*. If our wording slightly differs from the original wording of Adorno, Adorno's wording is quoted in italics after the form we used. There were 5 response categories: completely agree, agree, neither agree nor disagree, disagree or completely disagree.

1. People can be divided into two different classes: the weak and the strong. (26)

2. If people would talk less and work more, everybody would be better off. (37)

3. Most people disappoint when you get to know them better (43) Familiarity breeds contempt. -Slightly altered because the original formulation is not translatable in Dutch-

4. Young people sometimes get rebellious ideas, but as they grow up they ought to get over them and settle down. (21)

5. Most of our social problems would be solved if we could somehow get rid of the immoral, crooked and freebleminded people. (34)

6. It's better to support those authorities who maintain order and prevent chaos, even if they are not so democratically minded. (22) It is best to use some prewar authorities in Germany to keep order and prevent chaos. - Slightly altered because the item is dated-

7. There is hardly anything lower than a person who does not feel a great love, gratitude and respect for his parents. (27)

8. Nowadays when so many different kinds of people move around and mix together so much, a person has to protect himself especially carefully against catching an infection or disease from them. (18)

9. Sex crimes, such as rape and attacks on children, deserve more than mere imprisonment; such criminals ought to be publicly whipped, or worse. (25)
10. The true way of life is disappearing so fast that force may be necessary to preserve it. (30) The original scale: The true AMERICAN way of life ..... 
11. The businessman and the manufacturer are much more important to society than the artist and the professor. (41) 

Reversed items 
12. Finally, most people do better than expected 
13. Weak people too should get an opportunity for a decent life. 
14. We should make an effort to give freebleminded people prospects is our society. 

Attitudes towards immigrants scale 
There were 5 response categories: completely agree, agree, neither agree nor disagree, disagree or completely disagree. 

1. Generally speaking, foreigners can't be trusted. 
2. Most Moroccan and Turks living in our country are decent people 
3. I would find it unpleasant to have foreign neighbours 
4. Foreign labourers come here to exploit our social security system. 
5. When you get to know the Turks better, most of them are friendly people. 
6. Foreigners are a threat to our culture and habits. 
7. If more than half of the students in my school were foreigners, I would go to another school. 
8. I would not mind to have a foreign neighbour. 

Appendix II 

PRELIS PROGRAM TO GENERATE 200 COVARIANCE MATRICES 

generating 200 covariance matrices by bootstrapping 
DA NI=11 
RA=a:\bledad.ruw 
OU MA=CM RS=200 SF=25 BM=a:\bledad.tmb 

LISREL® 8 PROGRAM TO GENERATE 200 LAMBDA (factorloadings) MATRICES 
estim 200 LX Matrices 
DA NI=11 NO=111 RP=200 
CM=c:\bledad.tmb 
MO NX=11 NK=1 
FR LX(1,1) LX(2,1) LX(3,1) LX(4,1) LX(5,1) LX(6,1) LX(7,1) LX(8,1) LX(9,1) 
LX(10,1) LX(11,1) 
OU LX=c:\bledad.LAM
Appendix III

LISREL® (Simplis) Program for the Test of Invariant Factor Structures for the Authoritarianism Scale, All Factor Loadings Constrained Equal

Windows LISREL® 8.03
By
Karl G Joreskog and Dag Sorbom

This program is published exclusively by Scientific Software International, Inc.,
1525 East 53rd Street - Suite 906
Chicago, Illinois 60615, U.S.A.
Voice: (800)247-6113, Voice & Fax: (312)684-4979
Use of this program is subject to the terms specified in the Universal Copyright Convention.

The following lines were read from file a:\bledcomt.li8:

Group 1: Testing Equality Of Factor Structures
Model A: Factor Loadings, Error Variances Invariant
Observed variables
V66 V67 V68 V69 V70 V72 V73 V74 V75 V76
Covariance Matrix From file a:\bledad1.cov
Sample Size = 103
Latent Variables: Autor
Relationships
V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 = Autor
Group 2: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad2.cov
Sample Size = 138
Group 3: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad3.cov
Sample Size = 120
Group 4: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad4.cov
Sample Size = 108
End of Problem
LISREL8 (SIMPLIS) PROGRAM FOR THE TEST OF INVARIANT FACTOR STRUCTURES FOR THE AUTHORITARIANISM SCALE, ALL FACTOR LOADINGS FREE TO ESTIMATE WITHIN EACH GROUP

WINDOWS LISREL 8.03

BY

KARL G JORESKOG AND DAG SORBOM

Group 1: Testing Equality Of Factor Structures
Model A: Factor Loadings, Error Variances Invariant
Observed variables
V66 V67 V68 V69 V70 V71 V72 V73 V74 V75 V76
Covariance Matrix From file a:\bledad1.cov
Sample Size = 103
Latent Variables: Autor
Relationships:
V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 = Autor
Group 2: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad2.cov
Sample Size = 138
Relationships:
V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 = Autor
Group 3: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad3.cov
Sample Size = 120
Relationships:
V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 = Autor
Group 4: Testing Equality Of Factor Structures
Covariance Matrix From File a:\bledad4.cov
Sample Size = 108
Relationships:
V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 = Autor
End of Problem